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Duopsony models with consistent conjectural variations

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In this note we develop a consistent conjectural variation model that generalizes Bresnahan's (1981) results to a duopoly-duopsony setting.[1] This is the first duopsony model in which firms are constrained to have consistent conjectural variations, and two interesting results emerge.[2] First, Bertrand conjectures occur when inputs are homogeneous and when input demand functions are horizontal. Second, the consistent conjectures equilibrium approaches Cournot when input demand functions are vertical or when the firm is a monopolist-monopsonist. This implies that firms will behave more competitively in a duopsony setting as inputs and outputs become more homogeneous and as input demand functions become relatively more elastic.

I. THE CONSISTENT CONJECTURAL VARIATIONS MODEL

Because firms with monopsony power may also have monopoly power, a rather general duopsony-duopoly model with consistent conjectural variations will be considered.[3] In terms of the output market, two firms (1 and 2) are assumed to compete in a single output market, where q_i denotes the output of firm 1 or 2. Entry barriers are sufficiently high so that entry is blocked.[4] Because firms 1 and 2 are generally assumed to be symmetrical, we shall focus on the behaviour of firm i and assume that firm j is the other firm when speaking about firm i. There are a sufficiently large number of buyers who generate the following inverse demand function for firm i:

$$p_i = p_i(q_i, q_j) \tag{1}$$

where p_i denotes the price output. Note that if output is homogeneous, then $p_i = p_j$ and $\partial p_i/\partial q_i = \partial p_i/\partial q_j$, and if output is differentiated, then $p_i \neq p_j$ and $\partial p_i/\partial q_i \neq \partial p_i\ \partial q_j$.

In the input market, the same two firms are the only firms that employ input x. For simplicity, these firms are assumed to use just one input[5] and face the same production function when inputs are assumed to be homogeneous

$$q_i = f(x_i) \tag{2}$$

There are many input suppliers who generate the following input supply function

$$w_i = w_i(x_i, x_j), \tag{3}$$

where w_i denotes the price of x_i. Note that if the inputs are homogeneous, then $w_i = w_j$ and $\partial w_i/\partial x_i = \partial w_i/\partial x_j$, and if the inputs are differentiated, then $w_i \neq w_j$ and $\partial w_i/\partial x_i \neq \partial w_i/\partial x_j$.

Firm i's problem is then to choose x in order to maximize profit, which is given by

[1]Although this modelling approach has its critics (Robson (1983), Lindh (1992)), its primary advantage is that, unlike game theory (Demsetz (1989), Fisher (1989), and Shepherd (1990)), it is easy to empirically test and leads to several interesting and testable implications. See Bresnahan (1989) and Schmalensee (1990) for a discussion of procedures to estimate output first order conditions (or output conjectural variations) and Schroeter (1988) and Chang and Tremblay (1991) for procedures to estimate input first order conditions (or input conjectural variations).
[2]Schroeter (1988) and Chang and Tremblay (1991) developed oligopsony models using the conjectural variation approach but did not consider the case where conjectural variations are consistent. Several have developed consistent conjectural variation models for output markets. For example, see Bresnahan (1981), Laitner (1980), Ulph (1983), and Shaffer (1991). According to Pace and Gilley (1990), this model was first developed by Leontief (1936).
[3]Robinson (1934, p. 227) states that, 'The most important cases of monopsony will occur in connection with monopoly.'
[4]See Perry (1982) for a discussion of a consistent conjectures output model with free entry.
[5]We do not generally assume a fixed proportions technology, which would constrain input and output conjectures to be equal and simply reproduce Bresnahan's (1981) results.

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$$\pi_i = p_i(q_i, q_j)q_i - w_i(x_i, x_j)x_i \quad (4)$$

To solve this problem, however, firm i must make some assumption about how its rival will react to a change in its own strategic variables. In this model, firm i is assumed to be concerned with j's input response to a change in x_i. Thus, firm i's first order (necessary) condition of profit maximization requires:

$$\partial\pi_i/\partial x_i = [(\partial p_i/\partial q_i)(\partial q_i/\partial x_i) + (\partial p_i/\partial q_j)(\partial q_j/\partial x_j)(\partial x_j/\partial x_i)]q_i + p_i(\partial q_i/\partial x_i) - [\partial w_i/\partial x_i + (\partial w_i/\partial x_j)(\partial x_j/\partial x_i)]x_i - w_i = 0 \quad (5)$$

For notational convenience, this condition will be rewritten as

$$\partial\pi_i/\partial x_i = p_i'q_i'q_i' + p_j'q_j'\nu_i q_i + p_i q_i' - w_i'x_i - w_j'\nu_i x_i - w_i = 0 \quad (6)$$

where $p_i' \equiv \partial p_i/\partial q_i$, $p_j' \equiv \partial p_i/\partial q_j$, $q_i' \equiv \partial q_i/\partial x_i$, $q_j' \equiv \partial q_j/x_j$, $w_i' \equiv \partial w_i/\partial x_i$, $w_j' \equiv \partial w_i/\partial x_j$, and $\nu_i \equiv \partial x_j/\partial x_i$. The term ν_i represents firm i's input conjectural variation (i.e., firm i's estimate of j's input reaction to a unit change in x_i).

Each firm's optimal input quantity cannot be determined without knowing the values of the conjectural variations. In our model, conjectural variations will be assumed to be consistent. By definition, a conjectural variation is consistent if it is equal to the optimal response of its rival at the equilibrium defined by that conjecture. For firm i this equals the change in x_j, as indicated by j's reaction function, in response to a unit change in x_i.

The consistent conjectures are calculated as follows. First, for simplicity it is assumed that the market demand function is negatively sloped and linear (at least locally), the input supply function is positively sloped and linear (locally), and the production function is quadratic, positive valued, increasing, and concave over positive input quantities.[6] At optimal values, firm i's first order condition is an identity and can be totally differentiated to obtain:

$$[p_i'q_i''q_i + p_i'q_i'q_i' + p_j'q_j'\nu_i q_i' + p_i'q_i'q_i' + p_i q'' - w_i' - w_j'\nu_i - w_i']dx_i + [p_j'q_j''\nu_i q_i + p_j'q_j'q_i' - w_j']\partial x_j = 0 \quad (7)$$

where $q_i'' \equiv \partial q_i'/\partial x_i$ and $q_j'' \equiv \partial q_j'/\partial x_j$. By rearranging terms, Equation 7 can be expressed as

$$\partial x_j/\partial x_j = \frac{-(p_j'q_j''\nu_i q_i + p_j'q_j'q_i' - w_j')}{p_i'q_i''q_i + p_i'q_i'q_i' + p_j'q_j'\nu_i q_i' + p_i'q_i - q_i' + p_i q_i'' - w_i' - w_j'\nu_i - w_i'} \quad (8)$$

This equation has three interpretations: (1) it equals the slope of firm i's input reaction function; (2) when evaluated at optimal values, it equals the true equilibrium change in x_i in response to a unit change in x_j and (3) it equals firm j's consistent input conjectural variation when solved for ν_j. Note that because of symmetry (i.e., firms 1 and 2 are assumed to face the same demand and technological conditions and behave the same), each firm's consistent conjectural variation will be the same; $\nu = \nu_1 = \nu_2$. Finally, a consistent conjectures equilibrium (CCE) exists if all conjectural variations are consistent, denoted by ν^*, and if each firm maximizes profits given the behaviour of its rival.

II. THE CONSISTENT CONJECTURES EQUILIBRIUM

This section will show how differing specifications of output demand, input supply and technology affect the CCE. First, assume homogeneous inputs and outputs and that each firm faces the same linear output demand function, linear production function, and linear input supply function. With these assumptions, Equation 8 simplifies to:

$$\nu_j = \frac{-[p'(q')^2 - w']}{p'(q')^2(2+\nu_i) - w'(2+\nu_i)} \quad (9)$$

The true consistent conjectural variations are obtained by recognizing that $\nu_i = \nu_j = \nu^*$ (given symmetry) and by solving Equation 9 for ν^*. Note that under these conditions, the Cournot equilibrium is not a CCE. A Cournot solution requires that $\nu_i = \nu_j = 0$, but when ν_i is set to 0 in Equation 9, ν_j equals $-1/2$.

This case parallels Bresnahan's (1981) Example 2 because it is the Bertrand equilibrium that is a CGE; $\nu^* = -1$. Because a nonlinear input supply function adds the term $(1 + \nu_i)q_i w''$ to both the numerator and the denominator of Equation 9, the input function need not be linear for Bertrand conjectures to be consistent. Generally however, (local) linearity is required in order for the slope of the reaction function to be a constant and for uniqueness.[7] Finally, note that when $\nu^* = -1$, the input market operates efficiently since the value of the marginal product for input x equals the price of input x from the first order condition.

Next, we relax the linear technology constraint by allowing the production function to be concave. With this added assumption Equation 8 becomes

$$\nu^* = \frac{-(p'q''\nu^*q + p'q'q' - w'}{p'q''q + pq'' + p'(q')^2(2+\nu^*) - w'(2+\nu^*)} \quad (10)$$

Again, Equation 10 shows that as the production function become linear ($q'' = 0$), the CCE approaches the Bertrand equilibrium. Furthermore, by solving for ν^* one can see that as the marginal product and input demand approach the vertical (q'' approaches ∞), ν^* approaches 0 and the input market CCE approaches Cournot.[8] This parallels Bresnahan's (1981) Example 3 and shows that the CCE departs from Bertrand and becomes less competitive as input demand functions become steeper.

[6]These assumptions simplify the analysis by ensuring that the consistent conjectural variations will be constants. See Bresnahan (1981) for further discussion.

[7]See Bresnahan (1981) for further discussion of this point in relation to output markets and the Appendix for a more complete derivation of the consistent conjectural variation and a discussion of uniqueness and the second order condition.

Finally, we will analyse the CCE when inputs and outputs are differentiated. For simplicity, we assume a linear production function. In this case, Equation 8 becomes

$$v^* = \frac{-(p_j' q_j' q_i' - w_j')}{2p_i'(q_i')2 + p_j' q_j' v^* q_i' - 2w_i' - w_j' v^*} \tag{11}$$

Again, the CCE is Bertrand only in the polar case where outputs and inputs are perfect substitutes (i.e., $p_i' = p_j'$ and $w_i' = w_j'$). At the other pole, Equation 11 implies that when outputs and inputs are not substitutable at all (p_j' and w_j' approach 0), then v^* approaches 0 and the model generates a monopoly-monopsony solution with just one firm.

The results of the model have several interesting implications. First, they parallel those of Bresnahan (1981), who found that under similar conditions the *output* CCE becomes less competitive as rival outputs become less substitutable and as marginal cost functions become steeper. In addition, this analysis helps to establish situations where Bertrand and Cournot input conjectures are sensible under the criteria established by Kreps (1990, pp. 443–9). For example, when the slope input demand functions approaches infinity, Cournot conjectures are sensible because it is appropriate for a firm to make x the strategic variable and assume that rivals' input quantities will remain fixed (a 0 input quantity conjectural variation). Alternatively, if inputs are homogeneous and input demand functions are horizontal, Bertrand conjectures are sensible because it would then be appropriate for the firm to make input price the strategic variable and assume that rivals' input prices will remain fixed (a 0 input price conjectural variation). Finally, if a firm is a monopolist and a monopsonist (i.e., the firm's output and inputs have no substitutes), then a Cournot conjecture is sensible because it generates the monopoly-monopsony solution.

III. CONCLUSION

In this note we develop a consistent conjectures model for a duopsony that leads to several interesting and testable implications. First, the model predicts that an input market will be less competitive when the inputs that are hired by each firm become more differentiated and when the input demand functions become relatively more inelastic. This result parallels Bresnahan's (1981) for a duopoly setting and is a rather 'intuitive theory of competition'[9] because one might expect product differentiation and the structure of input demand functions to affect input market performance in this way. In addition, our results clarify the conditions under which Bertrand and Cournot conjectures might occur. Bertrand conjectures are most likely when inputs are homogeneous and when input demand functions are horizontal, and Cournot conjectures are most likely when input demand functions are vertical or when the firm is a monopolist-monopsonist.

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[8]In this case, the second order condition of profit maximization requires that:

$$q''(p'q + p) + p'(q')^2(2 + \nu_i) - w'(2 + \nu_i) < 0.$$

This condition is satisfied if $\nu_i < 2$ [which is reasonable since a firm's conjectural variation is likely to lie between Bertrand ($\nu_i = -1$) and cartel ($\nu_i = 1$)], the output demand function is negatively sloped ($p' < 0$), the input supply function is positively sloped ($w' > 0$), and $p'q + p > 0$. Note that this last term can be rewritten as $p[1 - (1/^n)]$ [where $^n \equiv (dq/dp\ (p/q) > 0$] and will be positive if the equilibrium price is greater than zero and the firm operates in the elastic region of demand.

[9]This is the phrase used by Bresnahan (1981, p. 943) to describe the implications of his duopoly with consistent conjectures.

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APPENDIX

This appendix demonstrates that there is a unique consistent conjecture that satisfies the second order sufficient (necessary) condition (SOC) when inputs and outputs are homogeneous and when output demand, input supply, and the production function are linear. Uniqueness can be established as follows. Recall Equation 9

$$v_j = \frac{-[p'(q')^2 - w']}{p'(q')^2(2 + v_i) - w'(2 = v_i)}$$

which simplifies to

$$v_j = \frac{-1}{(2 + v_i)} \tag{A1}$$

Setting $v_i = v_j = v$ (by symmetry), Equation A1 can be rewritten as

$$v^2 + 2v + 1 = 0. \tag{A2}$$

The roots of this quadratic equation produce single consistent conjectural variation equal to –1.

Under the assumptions given above, the SOC of profit maximization is satisfied. To see this, note that profit maximization requires that

$$p'(q')^2(2 + v_i) - w'(2 + v_i) < 0. \tag{A3}$$

This condition is satisfied when $v_i = -1$ and the output demand and input supply functions are regular (i.e., $p' < 0$ and $w' > 0$, respectively).